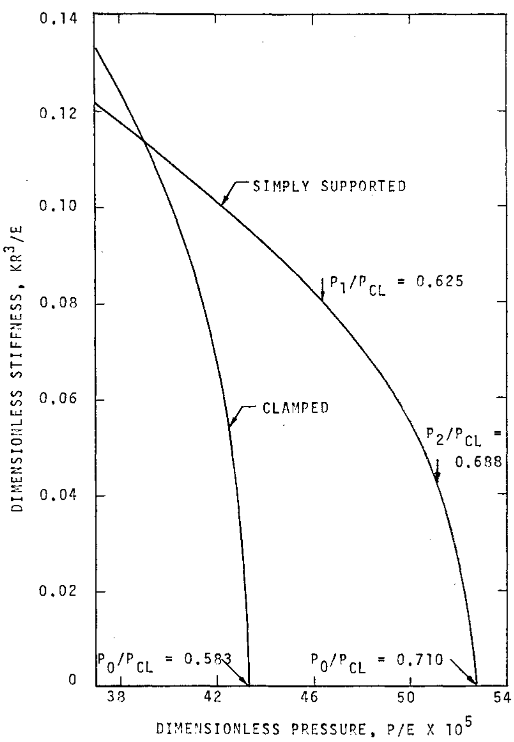

Fig. 1 Stiffness variation of shallow spherical shells (λ = 4) under uniform live pressure.

In addition to the above calculations, the $N = 1$ and $N = 2$ bifurcation pressures were computed for simply supported edges. In these calculations, linearization of the prebuckling state was made about the nonlinear prebuckling states corresponding to $p/p_{cl} = 0.60$ and 0.65 for the $N = 1$ and 2 buckling modes, respectively. As indicated in Fig. 1, the bifurcation pressures are $p_1/p_{cl} = 0.625$ and $p_2/p_{cl} = 0.688$. These compare with the $N = 2$ result of $p_2/p_{cl} = 0.65$ given in Ref. 3. Thus, as Stein suspected, the $N = 1$ buckling mode is critical for simply supported edges, but the associated critical pressure is still above the clamped shell buckling pressure.

In order to assess the effect of small geometric imperfections on these two buckling modes, the associated values of the second post-buckling coefficient b and the first imperfection parameter α were also computed. Based on buckling modes normalized to have their maximum normal deflection equal to the shell thickness, $b = -1.54$ for the $N = 1$ mode and $b = -1.06$ for the $N = 2$ mode. Also, for imperfections in the shape of the buckling modes, $\alpha = 0.581$ for the $N = 1$ mode and $\alpha = 0.386$ for the $N = 2$ mode. From these results and Eq. (78) of Ref. 5, it is clear that the $N = 1$ mode not only has a lower bifurcation pressure but is also more sensitive to small imperfections than the $N = 2$ mode. As an example, for an imperfection with a normal deflection amplitude of only one-tenth of the shell thickness, Eq. (38) of Ref. 5 gives a buckling load knockdown factor for the $N = 1$ mode of 0.672. Thus, for this very small imperfection, the shell would snap at a pressure of only 42% of the classical value for the complete spherical shell. It is interesting to note that this value is in rather good agreement with the experimental results shown for $\lambda = 4$ in Ref. 1 and 2.

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Effect of Real Gas Properties on the Base Pressure of a Blunt-Nosed Vehicle

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THE purpose of this Note is to point out the possibility that the base pressure of a blunt-nosed vehicle flying through the atmosphere at high velocities may be influenced by real gas properties. The values of base pressure in flight should be higher than those obtained in cold flow wind tunnels, which are essentially perfect gas facilities. The effect should be large for blunt vehicles and small for sharp-nosed slender vehicles.

In order to illustrate the possible differences, the hypothetical blunt-nosed vehicle shape shown in Fig. 1 is assumed. This particular shape was chosen to accentuate the real gas effects. The re-entry trajectory of Fig. 1 was taken from Ref. 1. The conditions are such that one may assume a turbulent boundary layer and, as a consequence, a negligible variation of base pressure with Reynolds number. The conditions of the gas on the flare (F) can be obtained using the following assumptions: 1) the flare pressure ratio, P_F/P_S is the same in flight as in the tunnel, and 2) the entropy increase across the flare shock is negligible.

The assumed variation of flare pressure ratio with free-stream Mach number is given in the lower part of Fig. 2.

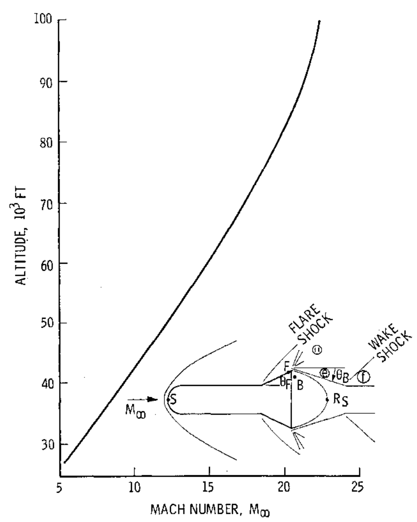

Fig. 1 Assumed model and trajectory.

Received June 13, 1969. The work reported herein was supported by the Arnold Engineering Development Center, Air Force Systems Command, under Contract F40600-69-C-0001 with ARO Inc. Further reproduction is authorized to satisfy the needs of the U.S. Government.

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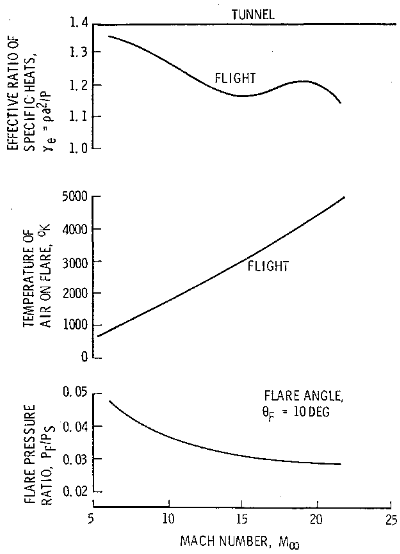

Fig. 2 Conditions on flare.

Using the altitude and Mach number, the stagnation-point conditions were obtained from Ref. 2. An isentropic expansion was then performed to the flare pressure using the data of Ref. 3. The middle portion of Fig. 2 gives the resulting static temperatures. The relatively high values of the static temperature assure that the gas does not behave as an ideal gas. This is also apparent from the values of the effective ratio of specific heats γ_e shown in the upper portion of Fig. 2.

To simplify matters, the expansion from the flare to the base is assumed to be a one-dimensional Prandtl-Meyer expansion. The gas properties are taken at the equilibrium values, so that the relationship between properties and the turning angle can be expressed as

$$d\theta_B = [(M^2 - 1)/U]^{21/2}dH$$

in which H is the static enthalpy, M the local Mach number, and U the flow velocity. The process is assumed isentropic.

There is no simple criterion to apply to relate the turning angle (θ_B) for the flight condition to the turning angle for the tunnel condition. The simplest assumption is that the turning angles are the same, but this lacks physical justification. In the absence of a detailed theory for the turbulent base flow region, a crude modeling parameter is used here. This parameter is based upon the Chapman model[4] of the base flow region in which the flow is brought to rest at the rear stagnation point (RS) because of the pressure increase across the wake shock. It is assumed that the primary forces acting on the base flow region are those due to the pressure difference across the wake shock and the viscous shear force due to the outer flow (e). Assuming that $P_B = P_e$ gives

$$(P_f - P_B)\pi R_B^2 \times \text{factors} = C_f \tfrac{1}{2}\gamma P_e M_e^2(\pi R_B^2/\tan\theta_B) \times \text{factors}$$

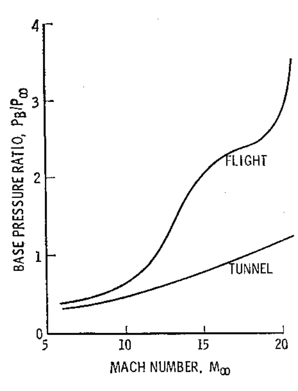

Fig. 3 Base pressure ratios in tunnel and in-flight.

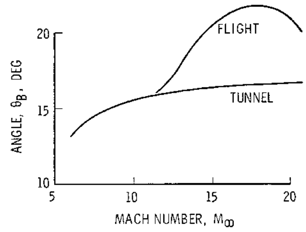

Fig. 4 Turning angle θ_B.

The factors on either side of the equation account for various geometric factors, and for deviations of the flow from the highly idealized model. From the simplified result, we find

$$(P_f/P_B - 1)\tan\theta_B/M_e^2 = KC_f$$

in which the factors have all been absorbed into the parameter K. This parameter may be assumed a function of Mach number, Reynolds number, and geometry. It is assumed here that these parameters are the same for flight and full scale, and therefore the right-hand side of the equation has the same value in-flight as in the tunnel. Actually there is usually a significant difference between the Reynolds numbers in-flight and in the tunnel. The preceding approach may still be used if the Reynolds numbers are high enough that base pressure is a weak function of Reynolds numbers. This occurs for a fully turbulent boundary layer.[5]

One further simplification is possible. Within the hypersonic small-disturbance approximation, the pressure after the wake shock (P_f) may be taken equal to the pressure after the flow has turned from the flare to the horizontal. Following the nomenclature of Ref. 5, this pressure is denoted by P''. Therefore,

$$(P''/P_B - 1)\tan\theta_B/M_e^2 = KC_f$$

This analysis points out the physical significance of the pressure ratio P_B/P'' used in Ref. 5.

From the base pressure ratio measured in the tunnel, the angle θ_B, the pressure ratio P''/P_B, and the Mach number M_e can be determined. The value of the parameter KC_f is therefore determined. For the flight condition, a Prandtl-Meyer expansion is made to an angle such that the same value is obtained for the parameter KC_f. This matching condition is a provisional one, and further study of this point is obviously necessary.

Results of calculations are shown in Fig. 3. The values of the base pressure ratio for the wind tunnel are estimated values, not actual values. It is seen that at the higher Mach numbers, the base pressure ratios in flight can be significantly higher than those from a cold flow wind tunnel.

The turning angles (θ_b) are shown in Fig. 4. The assumed matching conditions lead to larger values in flight than in the tunnel. If the flow had been turned through the same angle, the calculated base pressure in flight would be even higher than shown in Fig. 3.

It is possible for the energy in the vibrational modes to be frozen in the expansion. The resulting flight values would then be closer to the tunnel values than those shown in Fig. 3.

These preliminary calculations have shown that real gas properties can have a significant effect on the base pressure.

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Steady Cylindrical Expansion of a Monatomic Gas into Vacuum

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THE purpose of this Note is to re-examine certain aspects of steady flows into vacuum with cylindrical symmetry, the original treatment being given in Ref. 1. Previous theoretical work on the subject has dealt with the problem via certain models of the Boltzmann equation of the B-G-K type. Edwards and Cheng[1] and, in a more sophisticated way, Hamel and Willis[2] both followed this line of attack and in this respect the present Note says nothing new. The approximation procedures adopted in these papers were put on a firmer base by Freeman[3] by use of the method of matched asymptotic expansions. However, for the case of Maxwell molecules with cylindrical symmetry, the straightforward application of this method to the Boltzmann equation appeared to fail. It is the purpose herein to examine this case and to deduce an approximation procedure that will produce a uniformly valid solution for large r. In doing so it is of theoretical interest to see how the Chapman-Enskog expansion provides us with this uniform approximation.

The Boltzmann equation for steady flow with cylindrical symmetry can be written in nondimensional form as

$$\xi \frac{\partial f}{\partial r} + \frac{\eta^2}{r} \frac{\partial f}{\partial \xi} - \frac{\xi\eta}{r} \frac{\partial f}{\partial \eta} = AnT^{\beta}(F - f) \qquad (1)$$

where (r,θ,z) are cylindrical coordinates and (ξ,η,ζ) are components of molecular velocity in the directions of increasing (r,θ,z). f is the distribution function and F the local Maxwellian. For the moment, we will be dealing with the B-G-K collision model with a collision frequency dependent on temperature.[4] Also we are dealing with flows from near continuum sources and so in suitably nondimensionalized variables A is proportional to the inverse source Knudsen number and is assumed large. For $\beta \neq 0$ (non-Maxwell molecules) the problem can be approached using the method of matched asymptotic expansions; an inner (continuum) expansion breaks down when $r = 0(A^{(3/2)\beta})$, and an outer solution can be constructed[2] enabling the whole solution to be described. The region of validity of the inner expansion is found by balancing both sides of Eq. (1). This is equivalent to examining the Chapman-Enskog expansion for the Boltzmann equation and in particular its uniformity for large r. The two approaches are identical.

For $\beta = 0$, the case of Maxwell molecules, the full Boltzmann equation is modelled exactly, at least to the extent relevant to this Note, by Eq. (1). However, in this case the aforementioned procedure cannot be followed because the collision terms in (1) will never become as large as the convective terms. Nevertheless it can be shown via the Navier-Stokes equations that there is a logarithmic singularity at infinity in an expansion about the inviscid solution; therefore, how do we construct a uniformly valid solution for large r? The answer centers on the distinction between the continuum and inviscid solutions, and it will be shown that although the continuum solution (the Chapman-Enskog expansion) is uniformly valid, the inviscid solution (the Euler expansion) is not.

The Chapman-Enskog solution of the Boltzmann equation involves an asymptotic expansion of the distribution function in powers of some reference Knudsen number. The variables in this solution are the full temperature, density, and mass velocity of the gas, all of which depend on the reference Knudsen number itself. It is this choice of these variables which makes the Chapman-Enskog solution uniformly valid in this problem. In this context the true zeroth-order approximation would be the inviscid solution that is obtained by expanding all the thermodynamic variables in powers of the reference Knudsen number, keeping the radial variable of order one; it is this solution that is not uniformly valid. It is apparent in this problem that a correct zeroth-order approximation is obtained for the distribution function by providing a Maxwellian distribution with the correct parameters in the form of temperature, density and gas velocity. In view of these observations the method of strained coordinates will be shown to be a natural method of approach in finding a uniformly valid approximation to the distribution function and thermodynamic variables.

The Chapman-Enskog solution for this problem can be written

$$f = f_0(T,n,u)[1 + 0(1/A)]$$

where

$$T = T(r,A) \qquad (2)$$

$$n = n(r,A)$$

and

$$u = u(r,A)$$

are the temperature, density, and gas velocity. The variables have been nondimensionalized with respect to inviscid sonic values and A^{-1} is the Knudsen number arising from this scaling. r is the radial coordinate and f_0 is the Maxwellian distribution with T, n, and u as parameters. For the sake of brevity the details of the first-order term in Eq. (2) are omitted, but it is these terms that give rise to the Navier-Stokes equations when moments of the full distribution function are taken. These transport equations can be written

$$u^2 \frac{du}{dx} + \frac{2}{5}\left\{u\frac{dT}{dx} - uT - T\frac{du}{dx}\right\} = \frac{4}{3}A^{-1}u^2\left\{T\frac{d^2u}{dx^2} + \frac{du}{dx}\cdot\frac{dT}{dx} - uT - \frac{u}{2}\cdot\frac{dT}{dx}\right\} + 0(A^{-2})$$

$$u\frac{dT}{dx} + \frac{2Tu}{3} + \frac{2T}{3}\frac{du}{dx} = \frac{5}{3}A^{-1}u\left\{\frac{4}{3}T\left[\left(\frac{du}{dx}\right)^2 - u\frac{du}{dx} + u^2\right] + T\frac{d^2T}{dx^2} + \left(\frac{dT}{dx}\right)^2\right\} + 0(A^{-2}) \qquad (3)$$

where $x = \log r$. The density and pressure have been eliminated using the continuity and perfect gas relations. In these equations the temperature and gas velocity are functions of r and A as in Eq. (2).

The inviscid solution is the zeroth-order term obtained by expanding, n, T, and u in powers of A^{-1}. If the solution so constructed is denoted by a subscript 0, then the temperature, density, and velocity take the familiar implicit form

$$T_0 = 2(1 - u_0^2/4)$$

$$n_0 = (4/3)^{3/2}(1 - u_0^2/4)^{3/2} \qquad (4)$$

Received July 31, 1969.

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